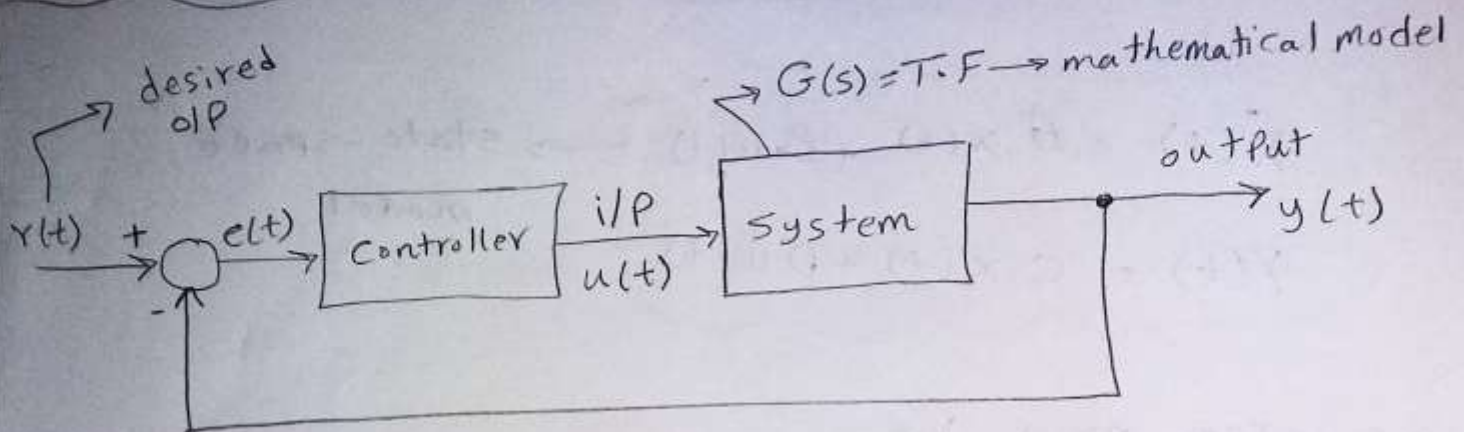


→ state space representation (model)



Controller

→ classical (conventional)

→ traditional

→ PID

Modern Controller

state-space model

→ mathematical model:-

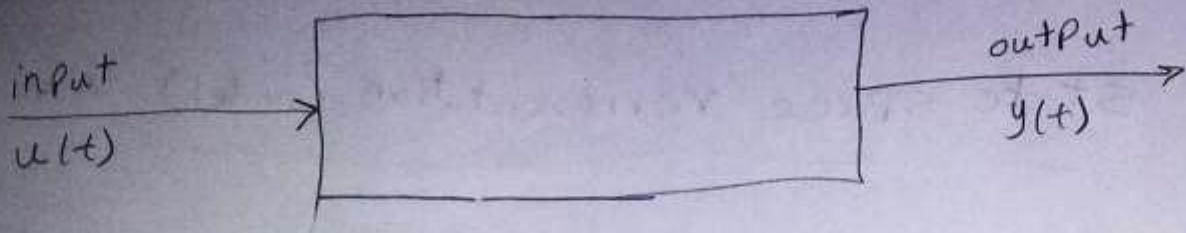
(1) T.F $G(s) = \frac{Y(s)}{U(s)}$

(2) Block diagram.

(3) signal flow graph.

(4) Differential eqn. [5] state-space model.

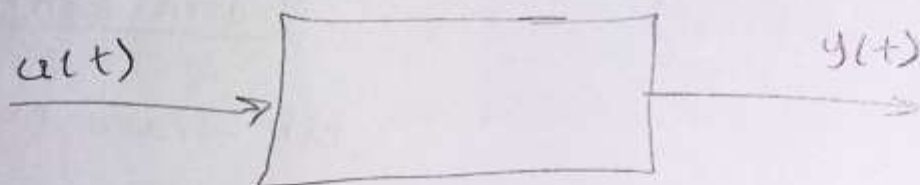
System = Process = Plant



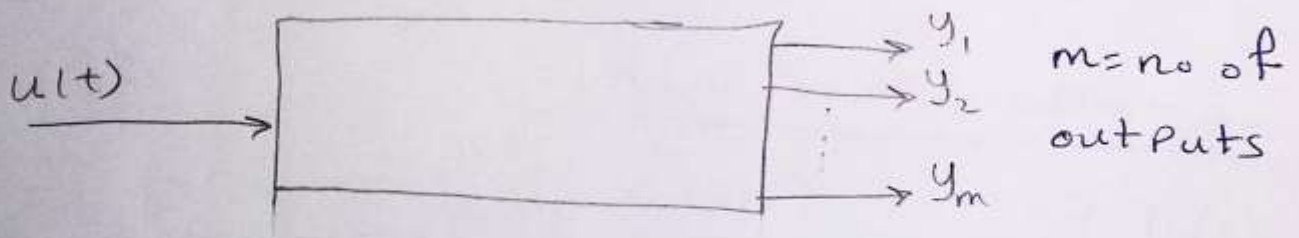
$$\begin{aligned} \dot{x}(t) &= A x(t) + B u(t) \\ y(t) &= C x(t) + D u(t) \end{aligned} \rightarrow \text{state-space model}$$

⇒ System may be

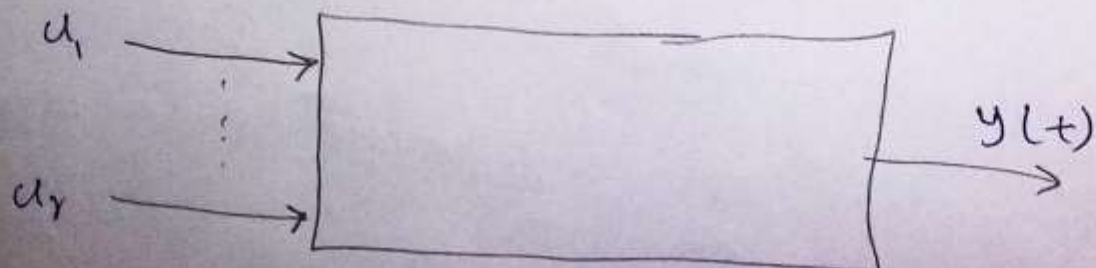
1) SISO ≡ single input single output



2) SIMO ≡ single input multi output.

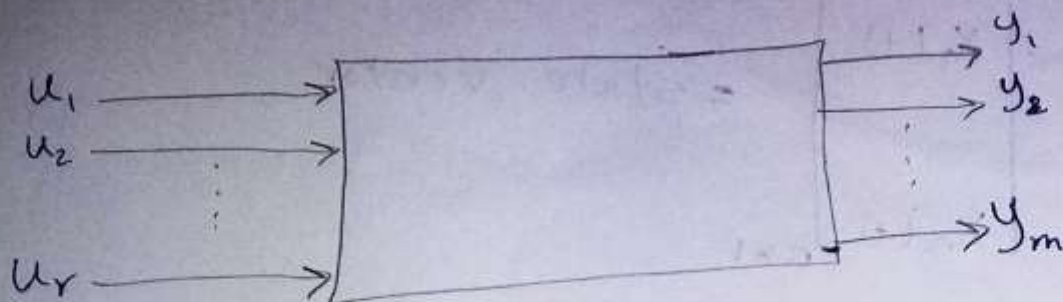


3) MISO ≡ multi input single output



$r \rightarrow$ no. of inputs.

4) MIMO



* if system is "SIMO" output will be:-

$$y(t) = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}$$

Column ~~vector~~ vector

مصفوفة بها عمود واحد

\Rightarrow m-dimensional output vector.

* For "MISO"

$$\begin{bmatrix} u_1 \\ \vdots \\ u_r \end{bmatrix}$$

$= u(t) \rightarrow$ input vector

* state variables:- ~~states~~

\rightarrow makes ^{it} ~~easy~~ easy to determine performance of behaviour of system.

$\rightarrow x_1(t), x_2(t), \dots, x_n(t)$

$n \rightarrow$ no. of state variables (states)

$$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}_{n \times 1} \quad \text{= state vector}$$

$$\dot{X}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \vdots \\ \dot{x}_n(t) \end{bmatrix}_{n \times 1}$$

→ we will deal with "SISO".

1) $A = \begin{bmatrix} \quad \end{bmatrix}_{n \times n}$ = state (system) matrix

2) $B = \begin{bmatrix} | \\ | \\ | \end{bmatrix}_{n \times 1}$ = input matrix.

3) $C = \begin{bmatrix} \text{---} \end{bmatrix}_{1 \times n}$ = output matrix.

4) $D_{1 \times 1}$ = scalar = direct transmission value
= feed forward value in vector-matrix form.

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \vdots \\ \dot{x}_n(t) \end{bmatrix}_{n \times 1} = \underbrace{\begin{bmatrix} & & \\ & & \\ & & \\ & & \\ & & \end{bmatrix}}_{A, n \times n} \begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix}_{n \times 1} + \underbrace{\begin{bmatrix} \\ \\ \\ \end{bmatrix}}_{B, n \times 1} * u(t)_{1 \times 1}$$

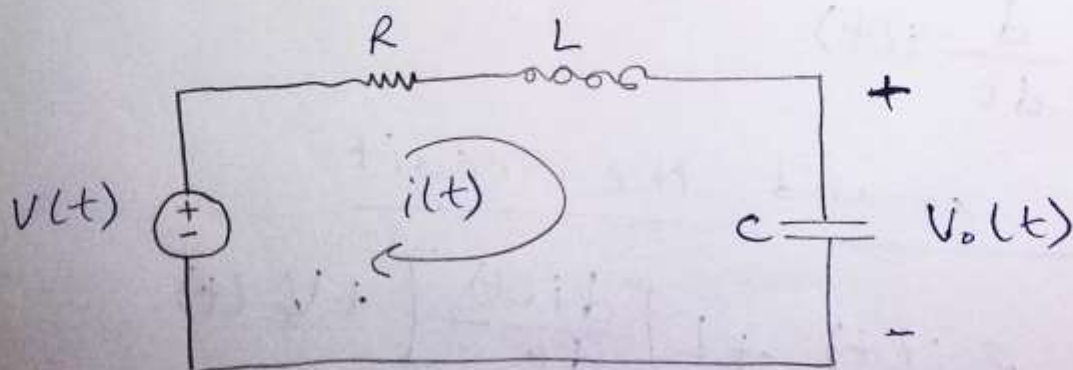
$$y(t)_{1 \times 1} = \underbrace{\begin{bmatrix} \\ \\ \\ \end{bmatrix}}_{C, 1 \times n} \begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix}_{n \times 1} + D_{1 \times 1} * u(t)$$

→ Dynamic eqns.

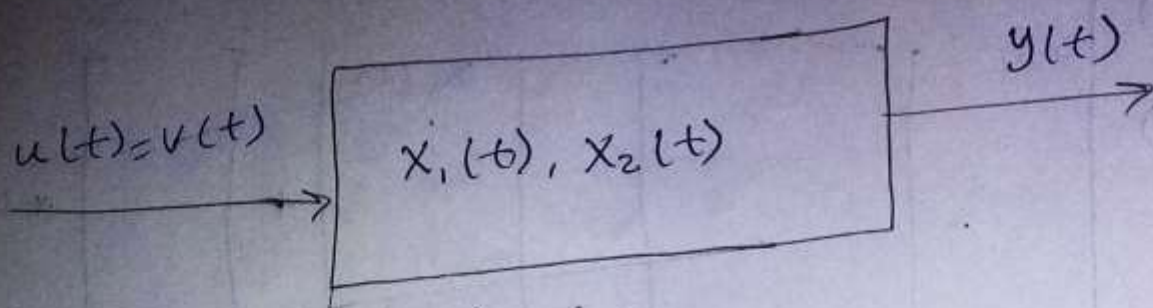
1) $\dot{x}(t) = A x(t) + B u(t)$ → state eqn.

2) $y(t) = C x(t) + D u(t)$ → output eqn.

→ Example (1) [Electrical system]



→ write state-space model.



$$\begin{aligned} X_1(t) &= i(t) \\ X_2(t) &= V_c(t) \end{aligned}$$

$n = \text{no. of states} = 2 \dots$

$$\begin{bmatrix} \dot{X}_1(t) \\ \dot{X}_2(t) \end{bmatrix}_{2 \times 1} = \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}_{2 \times 2} \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix}_{2 \times 1} + \begin{bmatrix} \cdot \\ \cdot \end{bmatrix}_{2 \times 1} u(t)_{2 \times 1}$$

$$y(t) = \begin{bmatrix} \cdot & \cdot \end{bmatrix}_{1 \times 2} \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix}_{2 \times 1} + D u(t)$$

$$V_L(t) = L \frac{di(t)}{dt}$$

$$i_c(t) = C \frac{dV_c(t)}{dt}$$

$$\dot{X}_1(t) = \frac{d}{dt} i(t)$$

→ KVL around the circuit

$$v(t) = R i(t) + L \left[\frac{di(t)}{dt} \right] + V_c(t)$$

$$\frac{di(t)}{dt} = \frac{-R}{L} i(t) - \frac{1}{L} V_c(t) + \frac{1}{L} V(t)$$

$$\dot{X}_1(t) = \frac{-R}{L} X_1(t) - \frac{1}{L} X_2(t) + \frac{1}{L} u(t) \quad \rightarrow (1)$$

$$\dot{X}_2(t) = \frac{d}{dt} V_c(t)$$

$$i(t) = C \frac{dV_c(t)}{dt} \Rightarrow \frac{dV_c(t)}{dt} = \frac{1}{C} i(t)$$

$$\therefore \dot{X}_2(t) = \frac{1}{C} X_1(t) \quad \rightarrow (2)$$

$$Y(t) = V_o(t) = V_c(t) = X_2$$

$$Y(t) = X_2(t) \quad \rightarrow (3)$$

$$\begin{bmatrix} \dot{X}_1(t) \\ \dot{X}_2(t) \end{bmatrix} = \begin{bmatrix} \frac{-R}{L} & \frac{-1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} u(t)$$

vectors are like \leftarrow

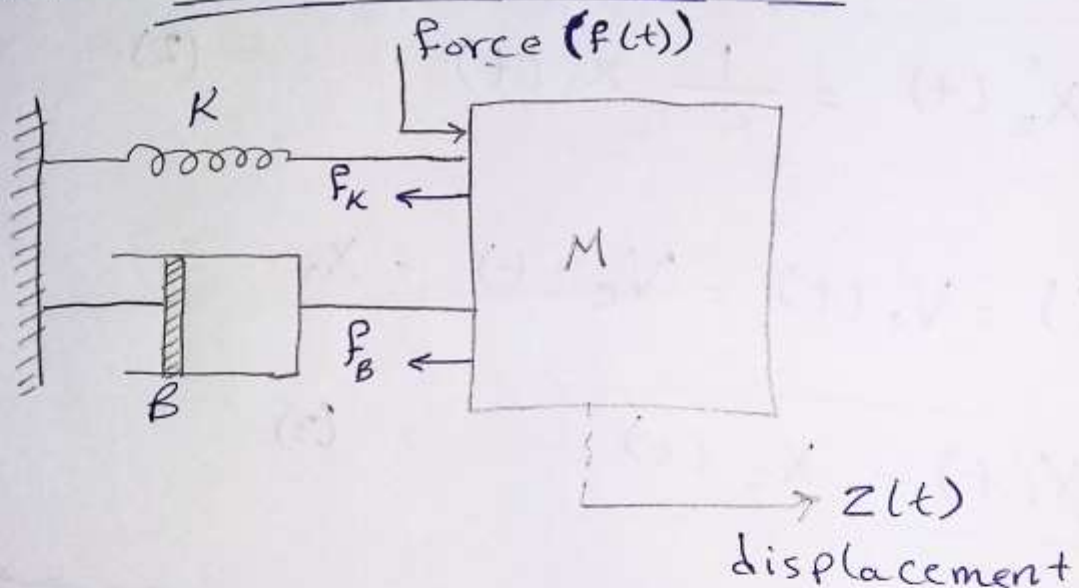
$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} + 0 \rightarrow Du(t)$$

← شكل المسائل ممكن يكون ←

$$\dot{X}(t)_{2 \times 1} = \underbrace{\begin{bmatrix} \frac{-R}{L} & \frac{-1}{L} \\ \frac{1}{C} & 0 \end{bmatrix}}_A X(t) + \underbrace{\begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}}_B u(t)$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} X(t)$$

Example (2) Mechanical system



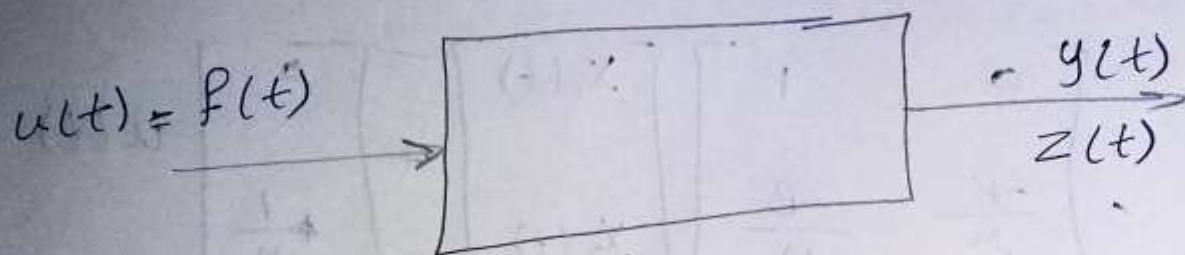
Find state space model

eqn of motion \Rightarrow differential eqn \Rightarrow
Newton's second law.

$$M \cdot \ddot{z}(t) = F(t) - B[\dot{z}(t) - 0] - K[z(t) - 0]$$

$$M \ddot{z}(t) + \beta \dot{z}(t) + K z(t) = F(t)$$

$n = \text{no. of state variables} \Rightarrow \begin{cases} x_1(t) \\ x_2(t) \end{cases}$



$\ddot{z} \rightarrow \text{acceleration}$. $\dot{z} \rightarrow \text{velocity}$.

$$x_1(t) = z(t) \quad x_2(t) = \dot{z}(t)$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} \quad \quad \quad \end{bmatrix}_{2 \times 2} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} \quad \quad \quad \end{bmatrix}_{2 \times 1} u(t)$$

$$y(t) = \begin{bmatrix} \quad \quad \quad \end{bmatrix}_{1 \times 2} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + D u(t)$$

$$x_1(t) = z(t) \Rightarrow \boxed{x_1(t) = x_2(t)} \rightarrow \textcircled{1}$$

$$\dot{x}_2(t) = \ddot{z}(t) = \frac{-K}{M} z(t) - \frac{\beta}{M} \dot{z}(t) + \frac{1}{M} F(t)$$

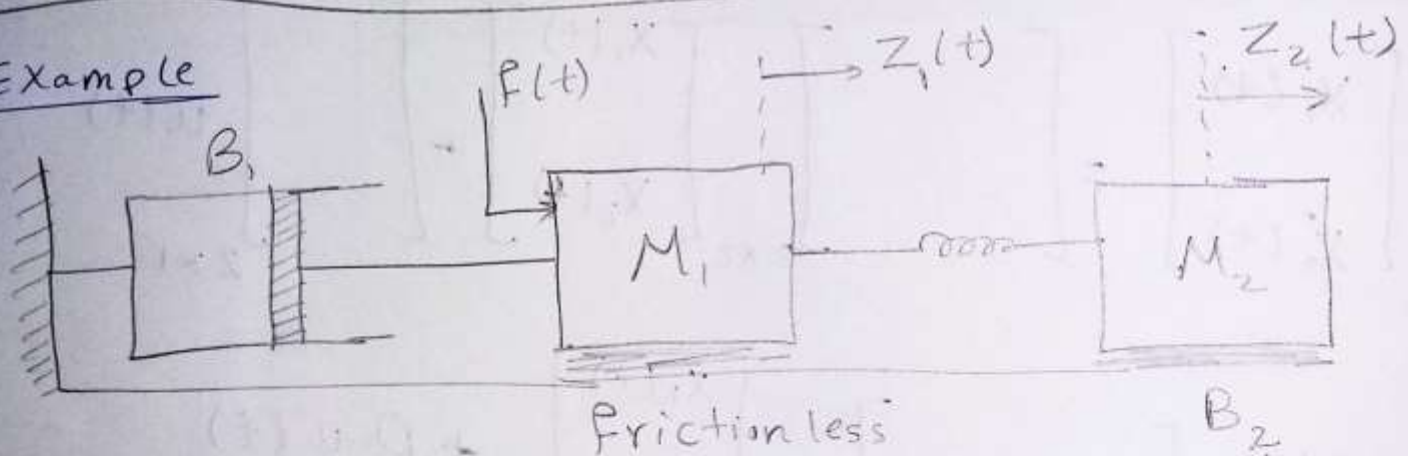
$$\dot{x}_2(t) = \frac{-K}{m} x_1(t) - \frac{B}{m} x_2(t) + \frac{1}{m} u(t) \quad \rightarrow (2)$$

output $y(t) = z(t) = x_2(t)$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-K}{m} & \frac{-B}{m} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + 0 u(t)$$

Example



* For Mass M_1

$$M_1 \ddot{z}_1(t) + B_1 \dot{z}_1(t) + K z_1(t) = F(t) + K z_2(t)$$

* For Mass M_2

$$M_2 \ddot{z}_2(t) + B_2 \dot{z}_2(t) + K z_2(t) = K z_1(t)$$

$$\boxed{n = 4}$$

$$\begin{array}{c} \downarrow Z_1 \\ Z_1(t), \dot{Z}_1(t) \end{array} \quad \begin{array}{c} \downarrow Z_2 \\ Z_2(t), \dot{Z}_2(t) \end{array}$$

$$X_1(t) = Z_1(t)$$

$$X_3(t) = Z_2(t)$$

$$X_2(t) = \dot{Z}_1(t)$$

$$X_4(t) = \dot{Z}_2(t)$$

$$\begin{bmatrix} \dot{X}_1(t) \\ \dot{X}_2(t) \\ \dot{X}_3(t) \\ \dot{X}_4(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-K}{M_1} & \frac{-B_1}{M_1} & \frac{K}{M_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K}{M_2} & 0 & \frac{-K}{M_2} & \frac{-B_2}{M_2} \end{bmatrix}_{4 \times 4} \begin{bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \\ X_4(t) \end{bmatrix}_{4 \times 1} + \begin{bmatrix} 0 \\ \frac{1}{M_1} \\ 0 \\ 0 \end{bmatrix}_{4 \times 1} u(t)$$

$$X(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}_{1 \times 4} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + 0 \cdot u(t)$$

$$\boxed{\dot{X}_1(t) = X_2(t)}$$

$$\dot{X}_2(t) = \frac{-K}{M_1} X_1 - \frac{B_1}{M_1} X_2(t) + \frac{K}{M_1} X_3(t) + \frac{1}{M_1} u(t)$$

$$\dot{X}_3(t) = X_4(t)$$

$$\dot{X}_4(t) = \frac{-K}{M_2} X_3 - \frac{B_2}{M_2} X_4 + \frac{K}{M_2} X_1$$

output

$$y(t) = Z_2(t) = X_3(t)$$

تم تحديد القيم في المصفوفة السابقة.

[12]

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t)$$